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Arrovian impossibilities in aggregating preferences over non-resolute outcomes

Emre Doğan · M. Remzi Sanver

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Abstract Let **A** be a set of alternatives whose power set is $\underline{\mathbf{A}}$. Elements of $\underline{\mathbf{A}}$ are interpreted as non-resolute outcomes. We consider the aggregation of preference profiles over $\underline{\mathbf{A}}$ into a (social) preference over $\underline{\mathbf{A}}$. In case we allow individuals to have any complete and transitive preference over $\underline{\mathbf{A}}$, Arrow's impossibility theorem naturally applies. However, the Arrovian impossibility prevails, even when the set of admissible preferences over $\underline{\mathbf{A}}$ is severely restricted. In fact, we identify a mild "regularity" condition which ensures the dictatoriality of a domain. Regularity is compatible with almost all standard extension axioms of the literature. Thus, we interpret our results as the strong prevalence of Arrow's impossibility theorem in aggregating preferences over non-resolute outcomes.

1 Introduction

We know since Arrow (1951) that given at least three alternatives, an aggregation rule that maps preference profiles into a preference cannot satisfy Pareto optimality,

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E. Doğan
Department of Economics, Rice University, İstanbul, Turkey

M. R. Sanver (✉)
Department of Economics, İstanbul Bilgi University, İstanbul, Turkey
e-mail: sanver@bilgi.edu.tr

pairwise independence[1] and non-dictatoriality simultaneously. This impossibility rests on a universal domain assumption which requires from the aggregation rule to be defined over all preference profiles.

Certain restrictions of the universal domain allow an escape from aggregation impossibilities. For example, the *single-peakedness* condition of Black (1948), the *single-cavedness* condition of Inada (1964) and the more general *value-restriction* condition of Sen (1966) lead to restricted domains over which the majority relation[2] is transitive. On the other hand, the Arrovian impossibility prevails over many other restricted domains. Such domains are qualified as *dictatorial*. Various conditions which render a domain of preference profiles dictatorial are identified by Blau (1957), Arrow (1963), Kalai and Muller (1977) Kalai, Muller and Satterthwaite (1979), Ozdemir and Sanver (2007).[3]

We consider a framework where individual preferences over the power set $\underline{\mathbf{A}}$ of a set of alternatives $\mathbf{A}$ are aggregated into a social preference over $\underline{\mathbf{A}}$. In case individuals are allowed to have any complete and transitive preference over $\underline{\mathbf{A}}$, the Arrovian impossibility naturally applies. However, one can assume a logical relationship between preferences over $\underline{\mathbf{A}}$ and preferences over $\mathbf{A}$. In other words, preferences over $\mathbf{A}$ can be axiomatically extended over $\underline{\mathbf{A}}$ which typically restricts the domain of preference profiles over $\underline{\mathbf{A}}$. We explore whether such restrictions circumvent the Arrovian impossibility. The answer is negative: We identify a mild "regularity" condition which ensures the dictatoriality of a domain. Moreover, regular domains are *superdictatorial*, i.e., exhibit the Arrovian impossibility in all of their superdomains.[4]

Of course, the "mildness" of regularity depends on the meaning attributed to a set. Throughout the paper, a set is interpreted as a list of mutually incompatible outcomes, i.e., a non-resolute outcome from which a single choice will eventually be made.[5] Regularity is perfectly compatible with this interpretation. In fact, as we discuss in Sects. 4 and 5, almost all extension axioms which conceive sets as non-resolute outcomes lead to regular (hence superdictatorial) domains. Thus, we interpret our results as the strong prevalence of Arrow's impossibility theorem in aggregating preferences over non-resolute outcomes.

Section 2 gives the basic notions. Section 3 proves the superdictatoriality of regular domains. Section 4 gives examples of regular domains. Section 5 makes some concluding remarks.

[1] known as *independence of irrelevant alternatives* or simply IIA.

[2] as defined in May (1952) and further explored by Asan and Sanver (2002) and Woeginger (2003).

[3] See Gaertner (2001) as well as Le Breton and Weymark (2003) for two excellent surveys of the literature.

[4] We know thanks to Bordes and Le Breton (1990) as well as Kelly (1994b) that the dictatoriality of a domain is not necessarily inherited by its superdomains.

[5] To be sure, there are other interpretations of a set, such as being

(i) a list of mutually compatible outcomes which are altogether chosen;
(ii) a menu from which the individual whose preference under consideration makes a choice;
(iii) a collection of states to which a (subjective) probability of occurrence is assigned.

All these interpretations have their own axioms. One can see Barberà et al. (2004) for a beautiful survey of this vast literature.

2 Basic notions

Consider a society $\mathbf{N}$ with $\#\mathbf{N} = n \geq 2$, confronting a set of alternatives $\mathbf{A}$ with $\#\mathbf{A} = m \geq 3$. Let $\underline{\mathbf{A}} = 2^{\mathbf{A}}\backslash\{\emptyset\}$ stand for the set of all non-empty subsets of $\mathbf{A}$. For each $k \in \{1, \ldots, m\}$, we write $\underline{\mathbf{A}}_k = \{X \in \underline{\mathbf{A}} : \#X = k\}$. We write $\Re$ for the set of complete and transitive binary relations over $\underline{\mathbf{A}}$. Every $i \in \mathbf{N}$ is assumed to have a *preference* $R_i \in \Re$ over $\underline{\mathbf{A}}$ whose strict counterpart is denoted P_i.[6] A *preference profile* (over $\underline{\mathbf{A}}$) is an n-tuple $\underline{R} = (R_1, \ldots, R_n) \in \Re^N$ of individual preferences.

A *domain* is any $D \subseteq \Re$ with $\#D \geq 2$. A *social welfare function* (SWF) defined over D is a mapping $\alpha : D^{\mathbf{N}} \to \Re$.[7] For any $\underline{R} \in D^{\mathbf{N}}$, we let $\alpha^*(\underline{R})$ stand for the strict counterpart of $\alpha(\underline{R})$.[8] A SWF $\alpha : D^{\mathbf{N}} \to \Re$ is *Pareto optimal* (PO) iff given any $X \in \underline{\mathbf{A}}$, any $Y \in \underline{\mathbf{A}}\backslash\{X\}$ and any $\underline{R} \in D^N$ with XP_iY for all $i \in \mathbf{N}$, we have $X\alpha^*(\underline{R})$ Y. A SWF $\alpha : D^{\mathbf{N}} \to \Re$ is *independent of irrelevant alternatives* (IIA) iff given any $X \in \underline{\mathbf{A}}$, any $Y \in \underline{\mathbf{A}}\backslash\{X\}$ and any $\underline{R}, \underline{R}' \in D^{\mathbf{N}}$ with $XR_iY \Leftrightarrow XR'_iY$ for all $i \in \mathbf{N}$, we have $X\alpha(\underline{R})Y \Leftrightarrow X\alpha(\underline{R}')$ Y. We qualify a SWF which is PO and IIA as *Arrovian*. We say that $\alpha : D^{\mathbf{N}} \to \Re$ is *dictatorial* iff there exists $d \in \mathbf{N}$ such that given any $\underline{R} \in D^{\mathbf{N}}$, any $X \in \underline{\mathbf{A}}$ and any $Y \in \underline{\mathbf{A}}\backslash\{X\}$, we have $XP_dY \Leftrightarrow X\alpha^*(\underline{R})$ Y. We call α *non-dictatorial* (ND) whenever it is not dictatorial.

We qualify a domain D as *dictatorial* iff every Arrovian SWF defined over D is dictatorial. We say that D is *superdictatorial* iff D is dictatorial while every superdomain of D is also dictatorial.[9] An immediate consequence of Arrow's impossibility theorem is that the full domain $\Re$ is dictatorial. We ask whether it is possible to escape the Arrovian impossibility by restricting $\Re$ through natural axioms which extend preferences over $\mathbf{A}$ to preferences over $\underline{\mathbf{A}}$.

We write Π for the set of complete, transitive and antisymmetric binary relations over $\mathbf{A}$. Each $\rho \in \Pi$ stands for an individual preference over $\mathbf{A}$ whose strict counterpart is ρ^*.[10] An *extension axiom* ε, assigns to each $\rho \in \Pi$, some non-empty $\varepsilon(\rho) \subset \Re$ such that given any $\rho \in \Pi$, any $R \in \varepsilon(\rho)$ and any distinct $X, Y \in \underline{\mathbf{A}}$, we have XPY whenever $x\rho y\, \forall x \in X, \forall y \in Y$.[11] Every extension axiom ε induces a domain $\cup_{\rho\in\Pi}\varepsilon(\rho)$ of admissible preferences over $\underline{\mathbf{A}}$.

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[6] For any $X, Y \in \underline{\mathbf{A}}$, we interpret XR_iY as "X being at least as good as Y in view of individual i". So $X\ P_iY$ holds whenever XR_iY but not YR_iX – which is interpreted as "X being preferred to Y by individual i".

[7] So we consider SWFs whose domains are Cartesian products of some domain D.

[8] i.e., for all $X, Y \in \underline{\mathbf{A}}$, we have $X\alpha^*(\underline{R})$ Y iff $X\alpha(\underline{R})$ Y but not $Y\alpha(\underline{R})$ X.

[9] Not every dictatorial domain has to be superdictatorial – see Footnote 4.

[10] So for any $x, y \in \mathbf{A}$, we have $x\rho^*y$ iff $x\rho y$ and not $y\rho^*x$. In fact, as ρ is antisymmetric, $x\rho y \Rightarrow$ not $y\rho x$ whenever x and y are distinct. On the other hand, by completeness, $x\rho x$ holds for all $x \in \mathbf{A}$.

[11] We interpret $\varepsilon(\rho)$ as the set of admissible preferences over $\underline{\mathbf{A}}$ when ρ is the preference over $\mathbf{A}$. Remark that the definition of an extension axiom incorporates the condition used by Kelly (1977) in his analysis of strategy-proof social choice rules: If the worst element of a set X is at least as good as the best element of a set Y, then X must be ranked above Y. This is the *conditio sine qua non* of extending preferences over non-resolute outcomes. Note that it implies the ordering of singleton sets to coincide with the ordering of the respective alternatives.

3 The impossibility

Let $\beta_\rho(X) \in \mathbf{A}$ be the best element in $X \in \underline{\mathbf{A}}$ at $\rho \in \Pi$, i.e., $\beta_\rho(X)\rho\, x\,\forall x \in X$. Similarly, let $\omega_\rho(X) \in \mathbf{A}$ be the worst element in $X \in \underline{\mathbf{A}}$ at $\rho \in \Pi$, i.e., $x\rho\omega_\rho(X)\forall x \in X$. We write $h(\rho; k) \in \underline{\mathbf{A}}_k$ for the highest ranked $k \in \{1, \ldots, m\}$ elements of $\mathbf{A}$ according to $\rho \in \Pi$, i.e., $x\ \rho\ y\,\forall x \in h\,(\rho; k), \forall y \in \mathbf{A}\backslash h(\rho; k)$. Similarly, $l(\rho; k) \in \underline{\mathbf{A}}_k$ is the lowest ranked $k \in \{1, \cdots, m\}$ elements of $\mathbf{A}$ according to $\rho \in \Pi$, i.e., $x\ \rho\ y\,\forall x \in \mathbf{A}\backslash l(\rho; k), \forall y \in l(\rho; k)$. We say that an extension axiom ε is *regular* iff the following four conditions hold at each $\rho \in \Pi$:

(i) Given any $X, Y \in \underline{\mathbf{A}}$ with $\beta_\rho(X)\rho^*\beta_\rho(Y)$, there exists $R \in \varepsilon(\rho)$ such that XPY.
(ii) Given any $X, Y \in \underline{\mathbf{A}}$ with $\omega_\rho(X)\rho^*\omega_\rho(Y)$, there exists $R \in \varepsilon(\rho)$ such that XPY.
(iii) Given any $k \in \{1, \ldots, m\}$ and any $X, Y \in \underline{\mathbf{A}}_k$ with $\beta_\rho(X)\rho^*\beta_\rho(Y)$, there exists $R \in \varepsilon(\rho)$ such that XPY while $h(\rho; k)\ P\ Z\,\forall Z \in \underline{\mathbf{A}}_k\backslash\{h(\rho; k)\}$ and $ZPl(\rho; k)\,\forall Z \in \underline{\mathbf{A}}_k\backslash\{l(\rho; k)\}$.
(iv) Given any $k \in \{1, \ldots, m\}$ and any $X, Y \in \underline{\mathbf{A}}_k$ with $\omega_\rho(X)\rho^*\omega_\rho(Y)$, there exists $R \in \varepsilon(\rho)$ such that XPY while $h(\rho; k)\ PZ\,\forall Z \in \underline{\mathbf{A}}_k\backslash\{h(\rho; k)\}$ and $ZPl(\rho; k)\,\forall Z \in \underline{\mathbf{A}}_k\backslash\{l(\rho; k)\}$.

The four regularity conditions can be verbally expressed as follows:

(i) If the best element in X is better than the best element in Y, then X must be allowed to be ranked above Y. To be clear, it is not that X has to be ranked above Y. Simply, Y cannot be forced to be ranked above X.
(ii) If the worst element in X is better than the worst element of Y, then X must be allowed to be ranked above Y. Again, it is not that X has to be ranked above Y.
(iii) Let X and Y be of the same cardinality, say k. If the best element in X is better than the best element in Y, then X must be allowed to be to be ranked above Y at an ordering where the best k-element set is $h(\rho; k)$ and the worst k-element set is $l(\rho; k)$.
(iv) Let X and Y be of the same cardinality, say k. If the worst element in X is better than the worst element in Y, then X must be allowed to be ranked above Y at an ordering where the best k-element set is $h(\rho; k)$ and the worst k-element set is $l(\rho; k)$.

Given our interpretation of sets, regularity is a very mild condition. In fact, as we discuss in Sect. 4, it is satisfied by almost all standard extension axioms of the literature.

We call a domain *regular* whenever it is induced by a regular extension axiom. We now show that all regular domains are superdictatorial.

Theorem 3.1 *Every regular domain D is superdictatorial.*

Proof Take any regular domain D. We first introduce the concepts that we use in our proof.

A triple[12] $\{X, Y, Z\} \subset \underline{\mathbf{A}}$ is said to be *free* in D iff given any distinct $K, L, M \in \{X, Y, Z\}$, there exists $P \in D$ such that $KPLP$ M.[13]

Given a SWF $\alpha : D^{\mathbf{N}} \to \Re$, $d \in \mathbf{N}$ is said to be *decisive over a pair*[14] $\{X, Y\} \subset \underline{\mathbf{A}}$ iff $(XP_dY \Rightarrow X\alpha^*(\underline{R})\ Y)$ and $(YP_dX \Rightarrow Y\alpha^*(\underline{R})\ X)$ hold for all $\underline{R} \in D^{\mathbf{N}}$.

We know from Kalai et al. (1979) as well as from Ozdemir and Sanver (2007) that under Arrovian SCWs, for each free triple, there exists an individual who is decisive over all pairs belonging to this free triple. Moreover, free triples which are "connected" in a particular way[15] exhibit the same decisive individual. We formally express these previous findings in the following lemma which we state without proof.[16]

Lemma 3.0 *Take any Arrovian SWF $\alpha : D^{\mathbf{N}} \to \Re$, any integer $s > 1$ and any sequence $\{X_1, Y_1\}, \ldots, \{X_s, Y_s\}$ of pairs. If $\{X_i, Y_i\} \cup \{X_{i+1}, Y_{i+1}\}$ forms a free triple in D for each $i = 1, \ldots, s-1$ then there exists $d \in \mathbf{N}$ who is decisive over each pair $\{X, Y\} \subset \bigcup_{i\in\{1,\ldots,s\}}\{X_i, Y_i\}$.*

We prove the theorem through five lemmata about the domain D. We first show that sets of same cardinality exhibit a common decisive individual.

Lemma 3.1 *Let $\alpha : D^{\mathbf{N}} \to \Re$ be an Arrovian SWF. For each $k \in \{1, \ldots, m-1\}$, $\exists\ d(k) \in \mathbf{N}$ who is decisive over every pair $\{X, Y\} \subset \underline{\mathbf{A}}_k$.*

Proof of Lemma 3.1 Take any $k \in \{1, \ldots, m-1\}$ and any triple $\{X, Y, Z\} \subset \underline{\mathbf{A}}_k$. We first show that $\{X, Y, Z\}$ is free in D. As X and Z are distinct while $\#X = \#Z$, $\exists x \in X\backslash Z$. If $x \in Y$, then pick some $\rho \in \Pi$ with $\beta_\rho(\mathbf{A}) = x$ and $h(\rho; k) = X$. By condition (iii) of regularity, $\exists R \in D$ such that $XPYPZ$. If $x \notin Y$, then pick some $z \in Z\backslash Y$. First, consider the case where $z \notin X$. Pick some $\rho \in \Pi$ with $\omega_\rho(\mathbf{A}) = z$ and $h(\rho; k) = X$. By condition (iv) of regularity, $\exists R \in D$ such that $XPYPZ$. Now, consider the case where $z \in X$. Pick some $\rho \in \Pi$ with $\beta_\rho(\mathbf{A}) = x$ and $l(\rho; k) = Z$. By condition (iii) of regularity, $\exists R \in D$ such that $XPYPZ$. Therefore, $\{X, Y, Z\}$ is a free triple in D. Now, write all elements of $\underline{\mathbf{A}}_k$ as a sequence of pairs $\{X_1, Y_1\}, \ldots, \{X_s, Y_s\}$ such that for each $i = 1, \ldots, s-1$, $\{X_i, Y_i\}$ and $\{X_{i+1}, Y_{i+1}\}$ have precisely one element in common. As every triple $\{X, Y, Z\} \in \underline{\mathbf{A}}_k$ is free in D, $\{X_i, Y_i\} \cup \{X_{i+1}, Y_{i+1}\}$ forms a free triple for each $i = 1, \ldots, s-1$. Hence by Lemma 3.0, there exists $d(k) \in \mathbf{N}$ who is decisive over each pair $\{X, Y\} \subset \underline{\mathbf{A}}_k$. □

Lemma 3.2 *Given any Arrovian SWF $\alpha : D^{\mathbf{N}} \to \Re$, we have $d(1) = d(2)$ while $d(1)$ is decisive over every pair $\{X, Y\} \subset \underline{\mathbf{A}}_1 \cup \underline{\mathbf{A}}_2$.*

[12] By a triple, we mean three distinct elements of $\underline{\mathbf{A}}$.

[13] The free triple condition is central to the analysis of preference aggregation in restricted domains. In fact, we know from Blau (1957) and Arrow (1963) that the Arrovian impossibility prevails over domains where every triple is free. More subtleties regarding the definition of a free triple is discussed by Kelly (1994a) who shows that when indifferences are allowed, not all logically possible (weak) orderings of a triple is necessary to establish an Arrovian impossibility.

[14] By a pair, we mean two distinct alternatives.

[15] such as described by the *saturatedness* condition of Kalai et al. (1979) or the *essential saturatedness* condition of Ozdemir and Sanver (2007).

[16] For the proof, see Theorem 1 of Kalai et al. (1979) or Lemma 3.1 of Ozdemir and Sanver (2007).

Proof of Lemma 3.2 We first show that given any distinct $a, b, c \in \mathbf{A}$, the triple $\{\{a\}, \{c\}, \{a, b\}\} \subset \underline{\mathbf{A}}_1 \cup \underline{\mathbf{A}}_2$ and the triple $\{\{a\}, \{a, b\}, \{a, c\}\} \subset \underline{\mathbf{A}}_1 \cup \underline{\mathbf{A}}_2$ are free in D. To see this, note that for any $\rho \in \Pi$,

(i) $a\,\rho\,b\,\rho\,c \Rightarrow \{a\}P\{a, b\}P\{c\}\,\forall\,R \in D$ and $\exists R \in D$ with $\{a\}P\{a, b\}P\{a, c\}$
(ii) $a\,\rho\,c\,\rho\,b \Rightarrow \exists\,R \in D$ with $\{a\}P\{c\}P\{a, b\}$ and $\exists\,R \in D$ with $\{a\}\,P\{a, c\}\,P\{a, b\}$
(iii) $b\,\rho\,a\,\rho\,c \Rightarrow \{a, b\}P\{a\}P\{c\}$ and $\{a, b\}P\{a\}P\{a, c\}\forall R \in D$
(iv) $b\,\rho\,c\,\rho\,a \Rightarrow \exists\,R \in D$ with $\{a, b\}P\{c\}P\{a\}$ and $\exists R \in D$ with $\{a, b\}P\{a, c\}P\{a\}$
(v) $c\,\rho\,a\,\rho\,b \Rightarrow \{c\}P\{a\}P\{a, b\}$ and $\{a, c\}P\{a\}P\{a, b\}\forall R \in D$
(vi) $c\,\rho\,b\,\rho\,a \Rightarrow \{c\}P\{a, b\}P\{a\}\,\forall\,R \in D$ and $\exists R \in D$ with $\{a, c\}P\{a, b\}P\{a\}$.

As $\{\{a\}, \{c\}, \{a, b\}\}$ and $\{\{a\}, \{a, b\}, \{a, c\}\}$ are free in D, by Lemma 3.0,

- $\exists\,d \in \mathbf{N}$ who is decisive over the pairs $\{\{a\}, \{c\}\}$, $\{\{a\}, \{a, b\}\}$ and $\{\{c\}, \{a, b\}\}$, and
- $\exists\,d' \in \mathbf{N}$ who is decisive over the pairs $\{\{a\}, \{a, b\}\}$, $\{\{a\}, \{a, c\}\}$ and $\{\{a, b\}, \{a, c\}\}$.

As d is decisive over the pair $\{\{a\}, \{c\}\}$, by Lemma 3.1, we have $d = d(1)$. As d' is decisive over the pair $\{\{a, b\}, \{a, c\}\}$, again by Lemma 3.1, we have $d' = d(2)$. On the other hand, there cannot be two distinct decisive individuals over $\{\{a\}, \{a, b\}\}$, so d and d', thus $d(1)$ and $d(2)$ coincide. We complete the proof by showing that $d(1)$ is decisive over every pair $\{X, Y\}$ with $X \in \underline{\mathbf{A}}_1$, $Y \in \underline{\mathbf{A}}_2$. To see this, take any $X = \{a\} \in \underline{\mathbf{A}}_1$ and any $Y \in \underline{\mathbf{A}}_2$. Consider first the case where $a \in Y$, i.e., $Y = \{a, b\}$ for some $b \in \mathbf{A}\backslash\{a\}$. We have already shown that given any $c \in \mathbf{A}\backslash\{a, b\}$, the triple $\{\{a\}, \{a, b\}, \{a, c\}\}$ is free in D. Therefore, there exists $d \in \mathbf{N}$ who is decisive over the pair $\{\{a\}, \{a, b\}\}$. But d is decisive over the pair $\{\{a, b\}, \{a, c\}\}$ as well, thus $d = d(1)$ is decisive over the pair $\{X, Y\}$. Now consider the case where a $\notin Y$, i.e., $Y = \{b, c\}$ for some $b, c \in \mathbf{A}\backslash\{a\}$. We have already shown that the triple $\{\{a\}, \{c\}, \{b, c\}\}$ is free in D. Therefore, there exists $d \in \mathbf{N}$ who is decisive over the pair $\{\{a\}, \{b, c\}\}$. But d is decisive over the pair $\{\{a\}, \{c\}\}$ as well, thus $d = d(1)$ is decisive over the pair $\{X, Y\}$.

Lemma 3.3 *Given any Arrovian SWF $\alpha : D^{\mathbf{N}} \to \Re$ and any $k \in \{3, \ldots, m-1\}$, we have $d(k) = d(1)$ while $d(1)$ is decisive over every pair $\{X, Y\} \subset \underline{\mathbf{A}}_1 \cup \underline{\mathbf{A}}_k$.*

Proof of Lemma 3.3 If $m = 3$, then Lemma 3.3 vacuously holds, as there exists no $k \in \{3, \ldots, m-1\}$. Now let $m > 3$ and take any $k \in \{3, \ldots, m-1\}$. We first show that for any distinct $a, b \in \mathbf{A}$ and for any $X \in \underline{\mathbf{A}}_k$, the triple $\{\{a\}, \{b\}, X\}$ is free in D. As $\#X \geq 3$, there exists $c \in X\backslash\{a, b\}$, and for any $\rho \in \Pi$,

(i) $c\rho a\rho b \Rightarrow \exists R \in D$ with $XP\{a\}P\{b\}$
(ii) $c\rho b\rho a \Rightarrow \exists R \in D$ with $XP\{b\}P\{a\}$
(iii) $a\rho b\rho c \Rightarrow \exists R \in D$ with $\{a\}P\{b\}PX$
(iv) $a\rho x$ for all $x \in X$ and $c\rho b \Rightarrow \exists R \in D$ with $\{a\}PXP\{b\}$
(v) $b\rho a\rho c \Rightarrow \exists R \in D$ with $\{b\}P\{a\}PX$
(vi) $b\rho x$ for all $x \in X$ and $c\rho a \Rightarrow \exists R \in D$ with $\{b\}PXP\{a\}$.

Therefore, the triple $\{\{a\}, \{b\}, X\}$ is free in D.

We now show that, given any $X \in \underline{\mathbf{A}}_k$, there exist $a \in X$ and $Y \in \underline{\mathbf{A}}_k$ such that the triple $\{\{a\}, X, Y\}$ is free in D. To see this, take any $X \in \underline{\mathbf{A}}_k$. As $m \geq 4$ and 3

$\leq k \leq m-1$, there exist distinct $a, b, c, d \in \mathbf{A}$ such that $a, c, d \in X$ and $b \notin X$. Take some $Y \in \underline{\mathbf{A}}_k$ such that $a, b, d \in Y$ and $c \notin Y$. For any $\rho \in \Pi$,

(i) $a = \beta_\rho(\mathbf{A}), b = \omega_\rho(\mathbf{A}) \Rightarrow \exists R \in D$ with $\{a\}PXPY$
(ii) $a = \beta_\rho(\mathbf{A}), c = \omega_\rho(\mathbf{A}) \Rightarrow \exists R \in D$ with $\{a\}PYPX$
(iii) $a = \beta_\rho(X)$ and $b\rho a \Rightarrow \exists R \in D$ with $YP\{a\}PX$
(iv) $a = \omega_\rho(X)$ and $b = \beta_\rho(\mathbf{A}) \Rightarrow \exists R \in D$ with $YPXP\{a\}$
(v) $c = \beta_\rho(\mathbf{A})$ and $a = \beta_\rho(Y) \Rightarrow \exists R \in D$ with $XP\{a\}PY$
(vi) $c = \beta_\rho(\mathbf{A})$ and $a = \omega_\rho(Y) \Rightarrow \exists R \in D$ with $XPYP\{a\}$.

Therefore, given any $X \in \underline{\mathbf{A}}_k$, there exist $a \in X$ and $Y \in \underline{\mathbf{A}}_k$ such that the triple $\{\{a\}, X, Y\}$ is free in D. Now take any $X \in \underline{\mathbf{A}}_k$ and let $\{\{a\}, X, Y\}$ be the corresponding free triple. By Lemma 3.0, there exists $d \in \mathbf{N}$ who is decisive over the pairs $\{\{a\}, X\}$, $\{\{a\}, Y\}$ and $\{X, Y\}$ and by Lemma 3.1, we have $d = d(k)$. We have also shown that $\{\{a\}, \{b\}, X\}$ is a free triple. So again by Lemma 3.0, there exists $d' \in \mathbf{N}$ who is decisive over the pairs $\{\{a\}, \{b\}\}$, $\{\{a\}, X\}$ and $\{\{b\}, X\}$ while by Lemma 3.1, we have $d = d(1)$. As there cannot be two distinct decisive individuals, d and d', over the pair $\{\{a\}, X\}$, $d(1)$ and $d(k)$ must coincide.

We complete the proof by showing that $d(1)$ is decisive over every pair $\{X, Y\}$ with $X \in \underline{\mathbf{A}}_1$, $Y \in \underline{\mathbf{A}}_k$. To see this, take any $X = \{a\} \in \underline{\mathbf{A}}_1$ and any $Y \in \underline{\mathbf{A}}_k$. As for any distinct $a, b \in \mathbf{A}$ and for any $Y \in \underline{\mathbf{A}}_k$ the triple $\{\{a\}, \{b\}, Y\}$ is free in D, there exists $b \in \mathbf{A}$ such that the triple $\{\{b\}, X, Y\}$ is free in D. Hence by Lemma 3.0, there exists $d \in \mathbf{N}$ who is decisive over the pairs $\{\{b\}, X\}$, $\{\{b\}, Y\}$ and $\{X, Y\}$. As d is decisive over the pair $\{\{b\}, X\}$, by Lemma 3.1, we have $d = d(1)$.

Lemma 3.4 *Given any Arrovian SWF* $\alpha: D^{\mathbf{N}} \to \Re$ *and any distinct* $k, k' \in \{2, \ldots, m-1\}$, *we have* $d(k) = d(k') = d(1)$ *while* $d(1)$ *is decisive over every pair* $\{X, Y\} \subset \underline{\mathbf{A}}_k \cup \underline{\mathbf{A}}_{k'}$.

Proof of Lemma 3.4 Take any distinct $k, k' \in \{2, \ldots, m-1\}$. We know by Lemmas 3.2 and 3.3 that $d(k) = d(k') = d(1)$. To prove that $d(1)$ is decisive over every pair $\{X, Y\} \subset \underline{\mathbf{A}}_k \cup \underline{\mathbf{A}}_{k'}$, take any $X \in \underline{\mathbf{A}}_k$ and any $Y \in \underline{\mathbf{A}}_{k'}$. We first show that there exist $a \in \mathbf{A}$ such that $\{\{a\}, X, Y\}$ is free in D.

Consider first the case where $X \supset Y$ or $Y \supset X$. Assume, without loss of generality, $X \supset Y$. Take any $a \in \mathbf{A} \backslash X$ and any $b \in X \backslash Y$. For any $\rho \in \Pi$,

(i) $a = \beta_\rho(\mathbf{A})$ and $b\rho y \,\forall\, y \in Y \Rightarrow \exists R \in D$ with $\{a\}PXPY$
(ii) $a = \beta_\rho(\mathbf{A})$ and $b = \omega_\rho(\mathbf{A}) \Rightarrow \exists R \in D$ with $\{a\}PYPX$
(iii) $b = \omega_\rho(\mathbf{A})$ and $y\rho a \,\forall\, y \in Y \Rightarrow \exists R \in D$ with $YP\{a\}PX$
(iv) $a = \omega_\rho(\mathbf{A})$ and $y\rho b \,\forall\, y \in Y \Rightarrow \exists R \in D$ with $YPXP\{a\}$
(v) $b\rho a$ and $a\rho y \,\forall\, y \in Y \Rightarrow \exists R \in D$ with $XP\{a\}PY$
(vi) $b\rho y \,\forall\, y \in Y$ and $a = \omega_\rho(\mathbf{A}) \Rightarrow \exists R \in D$ with $XPYP\{a\}$.

Consider now the case where $Y \not\subset X$ and $X \not\subset Y$. We look at the two exhaustive subcases:

In the first subcase where $X \cap Y = \emptyset$, take any $a \in X$. For any $\rho \in \Pi$,

(i) $a = \beta_\rho(\mathbf{A}) \Rightarrow \exists R \in D$ with $\{a\}PXPY$
(ii) $a = \beta_\rho(\mathbf{A})$ and $y\rho x \forall y \in Y, \forall x \in X\backslash\{a\} \Rightarrow \exists R \in D$ with $\{a\}PYPX$

(iii) $a = \beta_\rho(X)$ and $y\rho a \forall y \in Y \Rightarrow YP\{a\}PX\forall R \in D$
(iv) $a = \omega_\rho(X)$ and $y\rho x \forall y \in Y, \forall x \in X \Rightarrow YPXP\{a\}\forall R \in D$
(v) $a = \omega_\rho(X)$ and $a\rho y \forall y \in Y \Rightarrow XP\{a\}PY\forall R \in D$
(vi) $y\rho a \forall y \in Y$ and $x\rho y \forall x \in X\backslash\{a\}, \forall y \in Y \Rightarrow \exists R \in D$ with $XPYP\{a\}$.

In the second subcase where $X \cap Y \neq \emptyset$, take any $b \in X\backslash Y$, any $c \in Y\backslash X$ and any $a \in X \cap Y$. For any $\rho \in \Pi$,

(i) $a = \beta_\rho(\mathbf{A})$ and $c = \omega_\rho(\mathbf{A}) \Rightarrow \exists R \in D$ with $\{a\}PXPY$
(ii) $a = \beta_\rho(\mathbf{A})$ and $b = \omega_\rho(\mathbf{A}) \Rightarrow \exists R \in D$ with $\{a\}YPX$
(iii) $c = \beta_\rho(\mathbf{A})$ and $a = \beta_\rho(\mathbf{A}\backslash\{c\}) \Rightarrow \exists R \in D$ with $YP\{a\}PX$
(iv) $c = \beta_\rho(\mathbf{A})$ and $a = \omega_\rho(X) \Rightarrow \exists R \in D$ with $YPXP\{a\}$
(v) $b = \beta_\rho(\mathbf{A})$ and $a = \beta_\rho(\mathbf{A}\backslash\{b\}) \Rightarrow \exists R \in D$ with $XP\{a\}PY$
(vi) $b = \beta_\rho(\mathbf{A})$ and $a = \omega_\rho(Y) \Rightarrow \exists R \in D$ with $XPYP\{a\}$.

Therefore, the triple $\{\{a\}, X, Y\}$ is free in D. So by Lemma 3.0, there exists a decisive individual over the pairs $\{\{a\}, X\}$, $\{\{a\}, Y\}$ and $\{X, Y\}$. We know by Lemmas 3.2 and 3.3 that $\{\{a\}, X\}$ and $\{\{a\}, Y\}$ have $d(1)$ as the decisive individual who is also decisive over $\{X, Y\}$. □

Lemma 3.5 *Given any Arrovian SWF $\alpha : D^{\mathbf{N}} \to \Re$, any $k \in \{1, \dots, m-1\}$ and any $X \in \underline{\mathbf{A}}_k$, $d(1)$ is decisive over $\{X, \mathbf{A}\}$.*

Proof of Lemma 3.5 Take any $k \in \{1, \dots, m-1\}$ and any $X \in \underline{\mathbf{A}}_k$. Consider first the case where $k = 1$, i.e., $X = \{x\}$ for some $x \in \mathbf{A}$. We will show that given any $a \in \mathbf{A}\backslash\{x\}$, the triple $\{\{a\}, \{x\}, \mathbf{A}\}$ is free in D. For any $\rho \in \Pi$,

(i) $a = \beta_\rho(\mathbf{A})$ and $x = \omega_\rho(\mathbf{A}) \Rightarrow \{a\}P\ \mathbf{A}\ P\{x\}\forall R \in D$
(ii) $a = \beta_\rho(\mathbf{A})$ and $\{x\} = \beta_\rho(\mathbf{A}\backslash\{a\}) \Rightarrow \exists R \in D$ with $\{a\}P\{x\}P\ \mathbf{A}$
(iii) $x = \omega_\rho(\mathbf{A})$ and $a = \omega_\rho(\mathbf{A}\backslash\{x\}) \Rightarrow \exists R \in D$ with $\mathbf{A}\ P\{a\}P\{x\}$
(iv) $a = \omega_\rho(\mathbf{A})$ and $x = \omega_\rho(\mathbf{A}\backslash\{a\}) \Rightarrow \exists R \in D$ with $\mathbf{A}\ P\{x\}P\{a\}$
(v) $x = \beta_\rho(\mathbf{A})$ and $a = \beta_\rho(\mathbf{A}\backslash\{x\}) \Rightarrow \exists R \in D$ with $\{x\}P\{a\}P\ \mathbf{A}$
(vi) $x = \beta_\rho(\mathbf{A})$ and $a = \omega_\rho(\mathbf{A}) \Rightarrow \{x\}P\ \mathbf{A}\ P\{a\}\forall R \in D$.

Therefore, the triple $\{\{a\}, X, \mathbf{A}\}$ is free in D. By Lemma 3.0, there exists $d \in \mathbf{N}$ who is decisive over the pairs $\{\{a\}, X\}$, $\{\{a\}, \mathbf{A}\}$ and $\{X, \mathbf{A}\}$ and by Lemma 3.1 we have $d = d(1)$. Thus, $d(1)$ is decisive over $\{\{x\}, \mathbf{A}\}$.

We now show that $d(1)$ is decisive over $\{X, \mathbf{A}\}$ when $\#X > 1$. Note that $\exists R \in D$ with $XP\{x\}P\ \mathbf{A}$ with $x \in X$. Take any $\underline{R} \in D^{\mathbf{N}}$ with $R_{d(1)} = R$. As $d(1)$ is decisive over the pair X, $\{x\}$ and the pair $\{x\}$, $\mathbf{A}$, we have $X\alpha^*(\underline{R})\{x\}$ and $\{x\}\alpha^*(\underline{R})\ \mathbf{A}$, which by transitivity implies $X\alpha^*(\underline{R})\ \mathbf{A}$. As α satisfies IIA, we have $X\alpha^*(\underline{R})\ \mathbf{A}$ whenever $XP_{d(1)}\ \mathbf{A}$. Now note that $\exists R \in D$ with $\mathbf{A}\ P\{x\}PX$ with $x \in X$. Take any $\underline{R} \in D^{\mathbf{N}}$ with $R_{d(1)} = R$. As $d(1)$ is decisive over the pair $\{\mathbf{A}, \{x\}\}$ and the pair $\{\{x\}, X\}$, we have $\mathbf{A}\ \alpha^*(\underline{R})\ \{x\}$ and $\{x\}\ \alpha^*(\underline{R})\ X$, which by transitivity implies $\mathbf{A}\ \alpha^*(\underline{R})\ X$. As α satisfies IIA, we have $\mathbf{A}\ \alpha^*(\underline{R})\ X$ whenever $\mathbf{A}\ P_{d(1)}\ X$. Thus, $d(1)$ is decisive over $\{X, \mathbf{A}\}$. □

The five lemmata above establish that $d(1)$ is decisive of any pair $\{X, Y\}$, proving the dictatoriality of D.

We showed that any regular domain D is dictatorial. Moreover, it is straightforward from the definition of regularity that any superdomain of a regular domain is also regular. Thus, any regular domain is superdictatorial, completing the proof of Theorem 3.1. □

4 Examples

We give examples of domain restrictions in the literature which are covered by Theorem 3.1.

4.1 The Kelly principle

This is an extension axiom κ used by Kelly (1977) in his analysis of strategy-proof non-resolute social choice rules. For every $\rho \in \Pi$, let $\kappa(\rho) = \{R \in \Re : \ XPY$ for all distinct $X, Y \in \underline{\mathbf{A}}$ with $x\rho y \forall x \in X, \forall y \in Y\}$. We write $D^{\kappa} = \cup_{\rho\in\Pi}\kappa(\rho)$ for the domain induced by κ.

4.2 The Gärdenfors principle

This is an extension axiom γ used by Gärdenfors (1976) in his analysis of strategy-proof non-resolute social choice rules. For every $\rho \in \Pi$, let $\gamma(\rho) = \{R \in \Re$: XPY for all distinct $X, Y \in \underline{\mathbf{A}}$ with $x\rho y \forall x \in X\backslash Y, \forall y \in Y\} \cap \{R \in \Re$: XPY for all distinct $X, Y \in \underline{\mathbf{A}}$ with $x\rho y \forall x \in X, \forall y \in Y\backslash X\} \cap \kappa(\rho)$. We write D^{γ} for the domain induced by γ.

4.3 Separability

This is an extension axiom σ used by Roth and Sotomayor (1990) in their manipulability analysis of many-to-one matching problems. For every $\rho \in \Pi$, let $\sigma(\rho) = \{R \in \Re : (X \cup \{x\})R(X \cup \{y\}) \Leftrightarrow x\rho y$ holds $\forall X \in 2^{\mathbf{A}}, \forall x, y \in \mathbf{A}\backslash X\} \cap \kappa(\rho)$. We write D^{σ} for the domain induced by σ.

4.4 Expected utility consistency

The literature on strategy-proof non-resolute social choice rules admit a variety of "expected utility consistency" definitions.[17] We confine ourselves to give a few examples.

Barberà et al. (2001) use in their analysis of strategy-proof non-resolute social choice rules, *conditional expected utility consistency* (CEUC) as an extension axiom. This is defined as follows:

A *utility function* is a real valued mapping u defined over $\mathbf{A}$. A utility function u *represents* $\rho \in \Pi$ iff $u(x) \geq u(y) \Leftrightarrow x\rho y \forall x, y \in \mathbf{A}$. A probability distribution over $X \in \underline{\mathbf{A}}$ is a mapping $\Omega^X : X \to [0, 1]$ with $\Sigma_{x\in X}\Omega^X(x) = 1$. We say that

[17] See Can et al. (2007) for an account of these.

$R \in \Re$ is CEUC with $\rho \in \Pi$ iff there exists a utility function u representing ρ and a probability distribution $\Omega : \mathbf{A} \to [0, 1]$ over $\mathbf{A}$ such that for all $X, Y \in \underline{\mathbf{A}}$, we have $XRY \Leftrightarrow \Sigma_{x \in X} \Omega^X(x)u(x) \geq \Sigma_{y \in Y} \Omega^Y(y)u(y)$ where for any $Z \in \underline{\mathbf{A}}$ and any $z \in Z$, we define $\Omega^Z(z) = \Omega(z)/\Sigma_{x \in Z}\Omega(x)$. For every $\rho \in \Pi$, let CEUC $(\rho) = \{R \in \Re : R$ is CEUC with $\rho\}$ and let D^{CEUC} be the domain induced by CEUC.

A second axiom used by Barberà et al. (2001) is *conditional expected utility consistency with equal probabilities* (CEUCEP) which is CEUC where Ω is restricted to be uniform. For every $\rho \in \Pi$, let CEUCEP $(\rho) = \{R \in \Re : R$ is CEUCEP with $\rho\}$ and let D^{CEUCEP} be the domain induced by CEUCEP.[18]

It is rather straightforward to check that D^{κ}, D^{γ}, D^{σ}, D^{CEUC}, D^{CEUCEP} are all regular domains which leads to the following corollary to Theorem 3.1:

Corollary 4.1 *D^{κ}, D^{γ}, D^{σ}, D^{CEUC}, D^{CEUCEP} are all superdictatorial domains.*

We wish to take the occasion to note that Barberà et al. (2001) establish the existence of a strategy-proof and non-dictatorial social choice correspondence over D^{CEUCEP} which we show to be dictatorial. Thus, D^{CEUCEP} exemplifies a case where the Arrovian impossibility holds but the Gibbard (1973) and Satterthwaite (1975) impossibility vanishes.

We close the section by considering lexicographic extensions which may escape Theorem 3.1.[19] For every $\rho \in \Pi$, let $\lambda^+(\rho) = \{R \in \Re : XPY$ for all $X, Y \in \underline{\mathbf{A}}$ with $\beta_\rho(X)\rho^*\beta_\rho(Y)\}$ and let $\lambda^-(\rho) = \{R \in \Re : XPY$ for all $X, Y \in \underline{\mathbf{A}}$ with $\omega_\rho(X)\rho^*\omega_\rho(Y)\}$. Write $D^{\lambda+}$ and $D^{\lambda-}$ for the domains that λ^+ and λ^- respectively induce. One can check that neither $D^{\lambda+}$ nor $D^{\lambda-}$ is regular while $D^{\lambda+} \cup D^{\lambda-}$ is regular.

5 Concluding remarks

While a *social choice correspondence* (SCC) maps preference profiles over alternatives into a set, a *social choice hyperfunction* (SCH) maps preference profiles over sets into a set. Hence, every SCC can be expressed in terms of a SCH.[20] As particular instances, SCHs are used by Barberà et al. (2001), Benoit (2002) and Ozyurt and Sanver (2006) in the analysis of strategy-proof social choice correspondences and by Ozkal-Sanver and Sanver (2006) in the exploration of Nash implementation via set-valued outcome functions. So there is a growing literature which conceives set-valued social choice rules as SCHs. Our model is a counterpart of this approach in the Arrovian aggregation framework, as the SWFs we consider naturally induce SCHs as social choice rules.

What makes our analysis of interest is the fact that extending preferences over sets, naturally restricts the domain of our SWFs. So it is worth asking whether

[18] In fact, D^{CEUCEP} coincides with the domain restriction considered by Fishburn (1972). Note also that both D^{CEUC} and D^{CEUCEP} are subsets of D^{κ}.

[19] See Pattanaik and Peleg (1984), Bossert (1995), Campbell and Kelly (2002), Kaymak and Sanver (2003) and Ozyurt and Sanver (2006) for explorations on lexicographic extensions with a variety of definitions.

[20] The converse statement is not true. One can see Barberà et al. (2001) as well as Ozyurt and Sanver (2006) for a discussion of this issue.

the Arrovian impossibility prevails under this additional structure. Our answer is strongly positive: Every regular domain is superdictatorial. We qualify this answer as strong, because regularity is a mild condition under our interpretation of a set. In fact, almost all the standard axioms of the literature which induce preferences over non-resolute outcomes, lead to regular domains. Among these, we have the extension axioms that Fishburn (1972), Pattanaik (1973), Gärdenfors (1976), Barberà (1977), Kelly (1977), Feldman (1979), Kannai and Peleg (1984), Roth and Sotomayor (1990), Barberà et al. (2001) and Kaymak and Sanver (2003) use in their analysis of preferences over non-resolute outcomes. Hence, we are able to announce the strong prevalence of the Arrovian impossibility when preferences over non-resolute outcomes are aggregated into a social preference.[21]

We close by noting that regularity does not exhibit a natural compatibility with interpretations of sets other than being non-resolute outcomes.[22] In fact, models which conceive a set as a list of mutually compatible alternatives[23] or a menu[24] or a collection of states[25] escape our Theorem 3.1. Nevertheless, we would not be surprised if impossibilities similar to the one established in this paper hold in these environments as well—a matter which we hope will be pursued.

References

Arrow K (1951) Social choice and individual values. Wiley, New York

Arrow K (1963) Social choice and individual values. Wiley, New York

Asan G, Sanver MR (2002) Another characterization of the majority rule. Econ Lett 75(3):409–413

Barberà S (1977) The manipulation of social choice mechanisms that do not leave "too much" to chance. Econometrica 45(7):1573–1588

Barberà S, Sonnenschein H, Zhou L (1991) Voting by committees. Econometrica 59(3):595–609

Barberà S, Dutta B, Sen A (2001) Strategy-proof social choice correspondences. J Econ Theory 101:374–394

Barberà S, Bossert W, Pattanaik PK (2004) Ranking sets of objects. In: Barberà S, Hammond PJ, Seidl C (eds.) Handbook of Utility Theory. Volume II Extensions, Kluwer Academic, Dordrecht, pp 893–977

Benoit JP (2002) Strategic manipulation in voting games when lotteries and ties are permitted. J Econ Theory 102:421–436

Black D (1948) On the rationale of group decision making. J Polit Econ 56:23–34

Blau JH (1957) The existence of social welfare functions. Econometrica 25:302–313

Bordes GA, le Breton M (1990) Arrovian theorems for economic domains: The case where there are simultaneously private and public goods. Soc Choice Welf 7:1–17

Bossert W (1995) Preference extension rules for ranking sets of alternatives with a fixed cardinality. Theory Decis 39:301–317

Campbell DE, Kelly JS (2002) A leximin characterization of strategy-proof and non-resolute social choice procedures. Econ. Theory 20:809–829

[21] As a positive result in this context, we have La Mura and Olschewski (2006) who interpret sets as decisions delegated to an external introduction. Their overcoming of the Arrovian impossibility is due to a restatement of the IIA condition within their environment.

[22] See Footnote 5.

[23] e.g., Barberà et al. (1991), Ozyurt and Sanver (2007).

[24] e.g., Kreps (1979), Gravel (1998), Dutta and Sen (1996), Nehring (1999), Dekel et al. (2001) and Gul and Pesendorfer (2001).

[25] Lainé et al. (1986) and Weymark (1997).

Can B, Erdamar B, Sanver MR (2007) Expected utility consistent extensions of preferences, Theory and Decision (in press)

Dekel E, Lipman BL, Rustichini A (2001) Representing preferences with a unique subjective state space. Econometrica 69(4):891–934

Dutta B, Sen A (1996) Ranking opportunity sets and Arrow impossibility theorems: correspondence results. J Econ Theory 71:90–101

Feldman A (1979) Non-manipulable multi-valued social decision functions. Public Choice 34:177–188

Fishburn PC (1972) Even-chance lotteries in social choice theory. Theory Decis 3(1):18–40

Gaertner W (2001) Domain conditions in social choice theory. Cambridge University Press, Cambridge

Gärdenfors P (1976) Manipulation of social choice functions. J Econ Theory 13:217–228

Gibbard A (1973) Manipulation of voting schemes: a general result. Econometrica 41:587–601

Gravel N (1998) Ranking opportunity sets on the basis of their freedom of choice and their ability to satisfy preferences: a difficulty. Soc Choice Welf 15(3):371–382

Gul F, Pesendorfer W (2001) Temptation and self-control. Econometrica 69(6):1403–1435

Inada K (1964) A note on the simple majority decision rule. Econometrica 32:525–531

Kalai E, Muller E (1977) Characterization of domains admitting non-dictatorial social welfare functions and non-manipulable voting procedures. J Econ Theory 16:457–469

Kalai E, Muller E, Satterthwaite MA (1979) Social welfare functions when preferences are convex, strictly monotonic and continuous. Public Choice 34:87–97

Kannai Y, Peleg B (1984) A note on the extension of an order on a set to the power set. J Econ Theory 32:172–175

Kaymak B, Sanver MR (2003) Sets of alternatives as Condorcet winners. Soc Choice Welf 20(3):477–494

Kelly JS (1977) Strategy-proofness and social choice functions without single-valuedness. Econometrica 45:439–446

Kelly JS (1994a) The free triple assumption. Soc Choice Welf 11:97–101

Kelly JS (1994b) The Bordes-Le Breton exceptional case. Soc Choice Welf 11:273–281

Kreps DM (1979) A representation theorem for "preference for flexibility". Econometrica 47(3):565–577

La Mura P, Olschewski G (2006) Non-dictatorial social choice through delegation. (unpublished manuscript)

Lainé J, Le Breton M, Trannoy A (1986), Group decision under uncertainty: a note on the aggregation of 'ordinal probabilities'. Theory Decis 21:155–161

Le Breton M, Weymark JA (2003) Arrovian social choice theory on economic domains. In: Arrow KJ, Sen AK, Suzumura K (eds) Handbook of social choice and welfare, vol 2, North-Holland, Amsterdam

May K (1952) A set of independent, necessary and sufficient conditions for simple majority decision. Econometrica 20:680–684

Nehring K (1999) Preference for flexibility in a Savage framework. Econometrica 67(1):101–119

Ozdemir U, Sanver MR (2007) Dictatorial domains in preference aggregation. Soc Choice Welf 28:61–76

Ozkal-Sanver I, Sanver MR (2006) Nash implementation via hyperfunctions. Soc Choice Welf 26(3):607–623

Ozyurt S, Sanver MR (2006) A general impossibility result on strategy-proof social choice hyperfunctions. (unpublished manuscript)

Ozyurt S, Sanver MR (2007) Strategy-proof resolute social choice correspondences. Soc Choice Welf (forthcoming)

Pattanaik PK (1973) On the stability of sincere voting situations. J Econ Theory 6:558–574

Pattanaik PK, Peleg B (1984) An axiomatic characterization of the lexicographic maximin extension of an ordering over the power set. Soc Choice Welf 1:113–122

Roth A, Sotomayor MAO (1990) Two-sided matching: a study in game theoretic modelling and analysis. Cambridge University Press, Cambridge

Satterthwaite MA (1975) Strategy-proofness and Arrow's conditions: existence and correspondence theorems for voting procedures and social welfare functions. J Econ Theory 10:187–217

Sen A (1966) A possibility theorem on majority decisions. Econometrica 34:491–499

Weymark JA (1997) Aggregating ordinal probabilities over finite sets. J Econ Theory 75:407–432

Woeginger G (2003) A new characterization of the majority rule. Econ Lett 81:89–94